OCR
Oxford Cambridge and RSA

Thursday 20 June 2024 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

QP

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

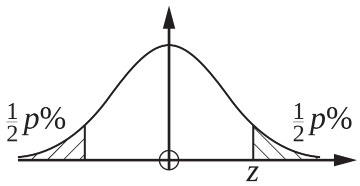
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A (60 marks)

1 Solve the inequality $\frac{x}{5} > 6 - x$. **[2]**

2 **(a)** The function $f(x)$ is defined by

$$f(x) = \sqrt{1+2x} \text{ for } x \geqslant -\tfrac{1}{2}.$$

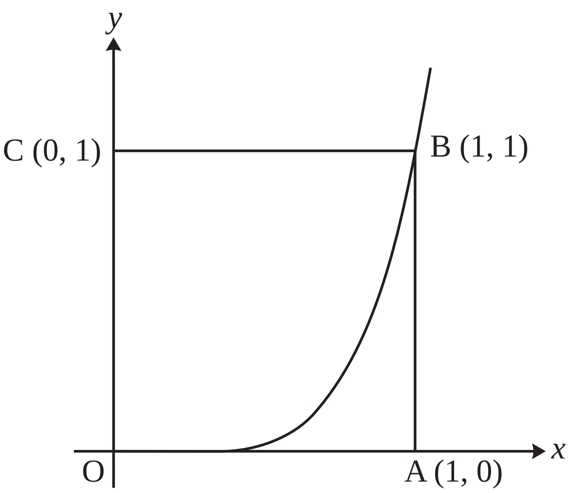
Find an expression for $f^{-1}(x)$ and state the domain of this inverse function. **[3]**

(b) Explain why $g(x) = 1 + x^2$, with domain all real numbers, has no inverse function. **[1]**

3 **In this question you must show detailed reasoning.**

The diagram shows the curve with equation $y = x^5$ and the square OABC where the points A, B and C have coordinates (1, 0), (1, 1) and (0, 1) respectively.

The curve cuts the square into two parts.



Show that the relationship between the areas of the two parts of the square is

$$\frac{\text{Area to left of curve}}{\text{Area below curve}} = 5.$$ **[4]**

4 **In this question you must show detailed reasoning.**

Determine the exact value of $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{2+\sqrt{3}}$. **[2]**

5 **In this question you must show detailed reasoning.**

Using the substitution $u = x + 1$, find the value of the positive integer c such that

$$\int_{c}^{c+4} \frac{x}{(x+1)^2}\,dx = \ln 3 - \frac{1}{3}.$$ **[6]**

6 **In this question you must show detailed reasoning.**

Solve the equation $\tan x - 3\cot x = 2$ for values of x in the interval $0° \leqslant x \leqslant 360°$. **[5]**

7 Prove that $\sin 8\theta \tan 4\theta + \cos 8\theta = 1$. **[3]**

8 **In this question you must show detailed reasoning.**

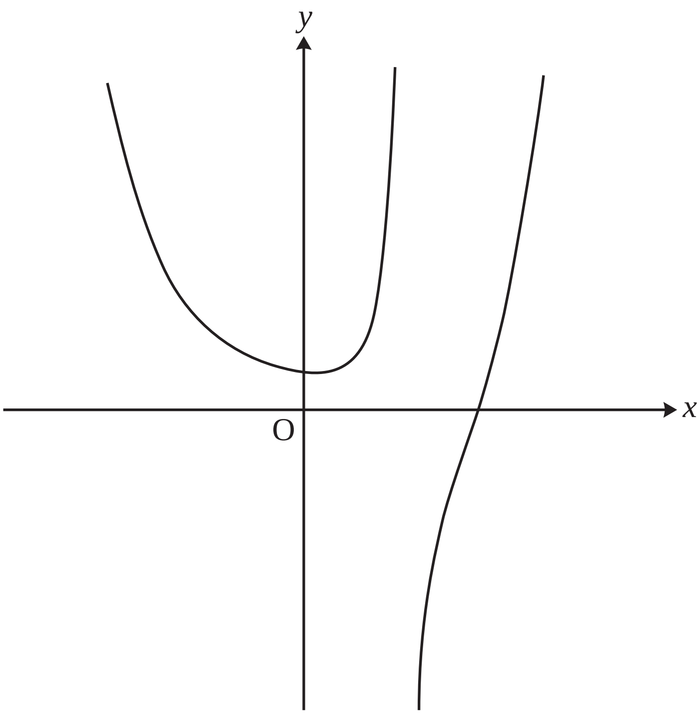
(a) Express $\cos x + \sqrt{3}\sin x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the values of R and α in exact form. **[4]**

(b) Hence solve the equation $\cos x = \sqrt{3}(1 - \sin x)$ for values of x in the interval $-\pi \leqslant x \leqslant \pi$. Give the roots of this equation in exact form. **[4]**

9 This question is about the equation $f(x) = 0$, where $f(x) = x^4 - x - \frac{1}{3x-2}$.

Fig. 9.1 shows the curve $y = f(x)$.

Fig. 9.1



(a) Show, by calculation, that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$. **[2]**

(b) **Fig. 9.2** shows part of a spreadsheet being used to find a root of the equation.

Fig. 9.2

	A	B
1	x	f(x)
2	1.5	3.1625
3	1.25	0.619977679
4	1.125	-0.250466087
5		

Write down a suitable number to use as the next value of x in the spreadsheet. **[1]**

(c) Determine a root of the equation $f(x) = 0$. Give your answer correct to **1** decimal place. **[1]**

(d) **Fig. 9.3** shows a similar spreadsheet being used to search for another root of $f(x) = 0$.

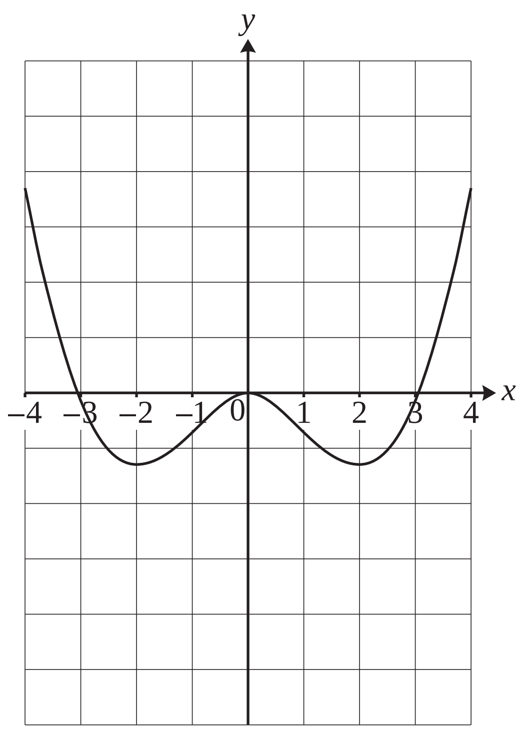
Fig. 9.3

	A	B
1	x	f(x)
2	0	0.5
3	1	-1
4	0.5	1.5625
5	0.75	-4.4336
6	0.6	4.5296
7	0.7	-10.4599
8	0.65	19.5285
9	0.675	-40.4674
10	0.6625	79.5301
11	0.66875	-160.4687

(i) Explain why it looks from rows 2 and 3 of the spreadsheet as if there is a root between 0 and 1. **[1]**

(ii) Explain why this process will **not** find a root between 0 and 1. **[1]**

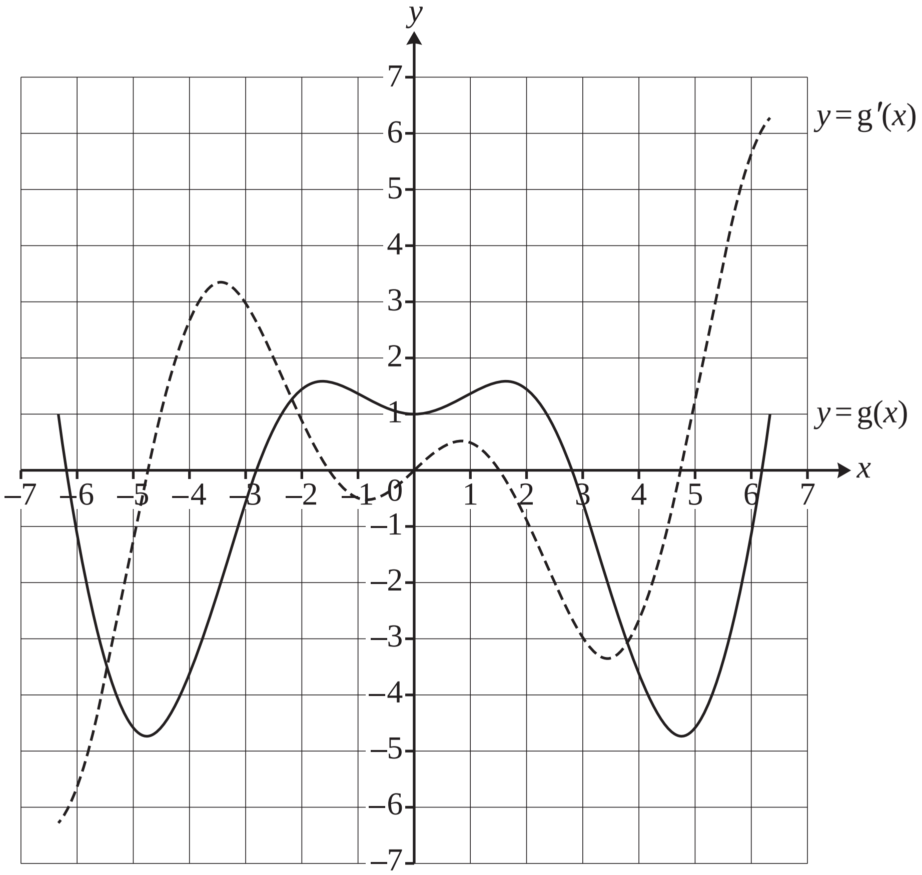
10 The diagram below shows the curve $y = f(x)$.



Sketch the graph of the gradient function, $y = f'(x)$, on the copy of the diagram in the **Printed Answer Booklet**. **[3]**

11 **Fig. 11.1** shows the curve with equation $y = \mathrm{g}(x)$ where $\mathrm{g}(x) = x\sin x + \cos x$ and the curve of the gradient function $y = \mathrm{g}'(x)$ for $-2\pi \leqslant x \leqslant 2\pi$.

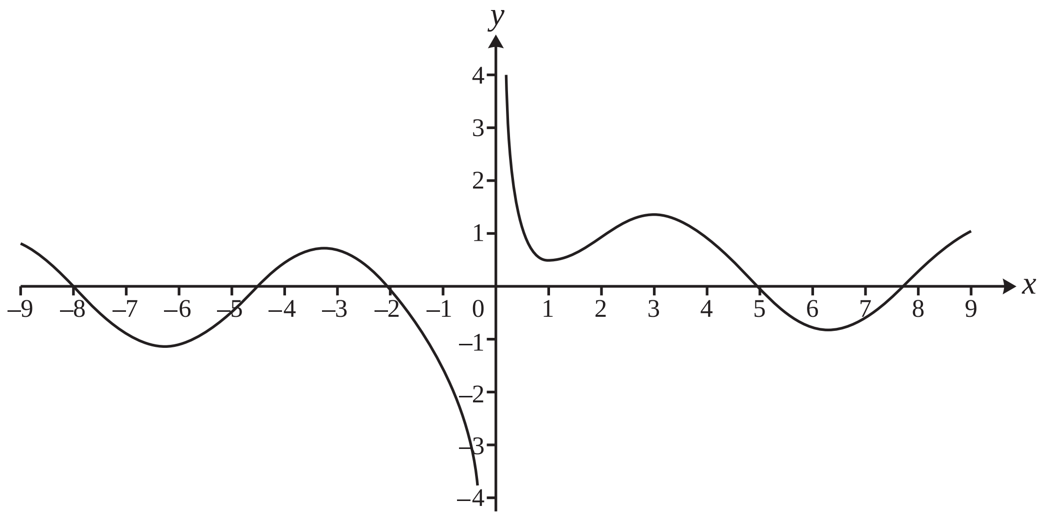
Fig. 11.1



(a) Show that the x-coordinates of the points on the curve $y = \mathrm{g}(x)$ where the gradient is 1 satisfy the equation $\frac{1}{x} - \cos x = 0$. **[3]**

Fig. 11.2 shows part of the curve with equation $y = \frac{1}{x} - \cos x$.

Fig. 11.2



(b) Use the Newton-Raphson method with a suitable starting value to find the smallest positive x-coordinate of a point on the curve $y = x\sin x + \cos x$ where the gradient is 1.

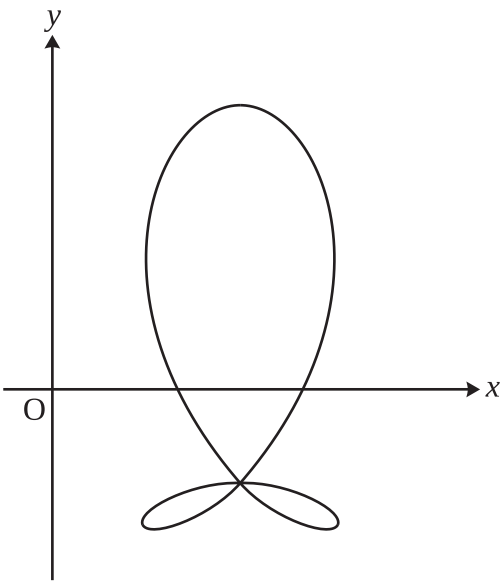
You should write down at least the following.

- The iteration you use
- The starting value
- The solution correct to **4** decimal places **[4]**

(c) Explain why $x_1 = 3$ is **not** a suitable starting value for the Newton-Raphson method in part **(b)**. **[1]**

12 The diagram shows the curve with parametric equations

$x = \sin 2\theta + 2,\ y = 2\cos\theta + \cos 2\theta$, for $0 \leqslant \theta < 2\pi$.



(a) **In this question you must show detailed reasoning.**

Determine the exact coordinates of all the stationary points on the curve. **[8]**

(b) Write down the equation of the line of symmetry of the curve. **[1]**

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

13 Substitute appropriate values of t_1 and t_2 to verify that $t_1 t_2$ gives the correct value for the y-coordinate of the point of intersection of the tangents at the points A and B in **Fig. C1**. **[1]**

14 Substitute appropriate values of t_1 and t_2 to verify that the expression $t_1^2 + t_2^2 + t_1 t_2 + \frac{1}{2}$ gives the correct value for the y-coordinate of the point of intersection of the normals at the points A and B in **Fig. C2**. **[1]**

15 **(a)** Show that, for the curve $y = ax^2 + bx + c$, the equation of the tangent at the point with x-coordinate t is $y = (2at + b)x - at^2 + c$. **[3]**

(b) Hence show that for the curve with equation $y = ax^2 + bx + c$, the tangents at two points, P and Q, on the curve cross at a point which has x-coordinate equal to the mean of the x-coordinates of points P and Q, as given in lines 11 to 14. **[3]**

16 Show that the expression $a\left(\frac{x_P + x_Q}{2}\right)^2 + b\left(\frac{x_P + x_Q}{2}\right) + c - a\left(\frac{x_P - x_Q}{2}\right)^2$ is equivalent to $ax_P x_Q + b\left(\frac{x_P + x_Q}{2}\right) + c$, as given in lines 15 and 16. **[2]**

17 Show that, for the curve $y = x^2$, the equation of the normal at the point (t, t^2) is $y = -\frac{x}{2t} + t^2 + \frac{1}{2}$, as given in line 27. **[3]**

18 A student is investigating the intersection points of tangents to the curve $y = 6x^2 - 7x + 1$. She uses software to draw tangents at pairs of points with x-coordinates differing by 5.

Find the equation of the curve that all the intersection points lie on. **[2]**

END OF QUESTION PAPER